INTEGRATED POOL/BILATERAL/RESERVE ELECTRICITY MARKET OPERATION UNDER PAY-AS-BID PRICING

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Abstract – A pricing model considering the simultaneous interaction of pool, bilateral and reserve markets in a power system is presented. The model is able to work under the classical marginal pricing (MP) approach or on the 'Pay-as-Bid' (PAB) pricing approach which is currently being considered as an alternative in some actual systems having most of their trades negotiated as bilateral contracts. In the PAB pricing approach, an integration process involves an AC Optimal Power Flow (OPF) for obtaining awarded bids in the pool and reserve market with the presence of long term firm bilateral contracts. As a result, the obtained prices of energy and reserve services incorporate the influences of the topology, voltage levels, losses, and capacity limits of generators and transmission lines. Results show that agents can plan their portfolios based on reasonable stable prices that reflect the impact of supplying several electricity services and associated costs of resources in several operation scenarios and bid strategies. From the perspective of the system regulator, the minimization of payments by PAB ensures the supply of energy, transmission losses and reserve requirements as well as enforcing financial adequacy. Numerical cases are presented for evaluating the model.

Keywords: Ancillary service, Pay as Bid, Pool, Bilateral and Reserve Markets, Average price

1 INTRODUCTION

In recent years, electricity markets have developed important economic and operational tools looking for efficiency in terms of determining prices that represent the costs of production and transmission of energy. From the structural point of view, there are two main forms of markets auctions for trading energy and ancillary services [1]. One is based on a separate private markets and the other is based on a strong coordination. In the first one, the provision of services is left to secondary markets after the main energy auction is defined. The possible advantage of this auction structure is that the voluntary participation of traders in markets could provide efficiency, like in a pool, avoiding using complex optimization tools. In the second kind of market auctions, electricity market products (energy and ancillary services) are procured simultaneously through central auctions. The advantage of this integrated market is that the resulting prices better reflect the cost of resources due to the inherent relationship between energy and ancillary services (like in systems as New York, New England and PJM).

From the theoretical point of view, two approaches are mainly followed for pricing electricity services: One of them is the classical marginal pricing (MP) where nodal prices represent the cost of the last MW to be supplied and the other is the "Pay-as-Bid" pricing (PAB) which is the way forward bilateral contracts are negotiated. Whether one or the other should be followed has been object of controversy as discussed in [6]. Recently, PAB pricing has showed an increasing interest because some markets are essentially based on bilateral agreements with agents motivated to minimize the volatility of nodal prices and the system regulator is also interested in obtaining financial adequacy [4,5].

This paper analyzes the characteristics of a pricing model designed for working under the classical Marginal Price (MP) and the Pay as Bid (PAB) strategies in a combined market structure involving the presence of long term forward physical bilateral contracts (not financial contracts), and short term trades like pool and ancillary reserve services. The purpose of the model is exploiting the advantages of centralized market coordination and the potential benefits of using the PAB pricing strategy. It is not the focus here to discuss which of the pricing approaches should be followed but instead observing their behavior in the combined market.

The ancillary reserve market is composed of several reserve services as presented in [1], and [2]. The characterization and requirements of these services are previously defined by the regulator and are based on the quality of their response (speed). The PAB version of this model is implemented through an integration process based on the Auman-Shapley [3,5] technique using an AC- OPF that takes into consideration the non-linear characteristic of the transmission network and allows unbundling the use and prices of several services.

The model characteristics allow market agents to plan their portfolios by knowing how prices of electricity services interact in several possible operation scenarios. From the ISO point of view, it is possible to estimate the impact of different levels and distributions of total bilateral trades, pool load and reserve requirements while enforcing price stability and financial adequacy.

The paper is organized as follows. Section 3 presents the formulation. Section 4 describes the pricing mechanism. Section 5 shows how revenues and payments are obtained. Section 6 describes the how to obtain generators and loads portfolios. Section 7 shows the reconciliation of costs. Section 8 shows numerical examples and Section 9 presents conclusions.

2 NOMENCLATURE

For each generator at bus i and demand at bus j participating in the integrated market we define the following notation.

Generators Bid Functions in ($/h):

$C_{g\,i}(p^p_{g\,i})$	Pool Energy
$C^{RU}_{g\,i}(ru_{g\,i})$	Regulation up reserve
$C^{RD}_{g\,i}(rd_{g\,i})$	Regulation down reserve
$C^{SR}_{g\,i}(sr_{g\,i})$	Spinning reserve
$C^{NS}_{g\,i}(ns_{g\,i})$	Non spinning reserve
$C^{RC}_{g\,i}(rc_{g\,i})$	Complementary reserve

Variables:

$p^p_{g\,i}$	Pool Active Generation awarded level (MW)
$ru_{g\,i}$	Awarded level of regulation up reserve (MW)
$rd_{g\,i}$	Awarded level of regulation down reserve (MW)
$sr_{g\,i}$	Awarded level of spinning reserve (MW)
$ns_{g\,i}$	Awarded level of non spinning reserve (MW)
$rc_{g\,i}$	Awarded level of complementary reserve (MW)
$q_{g\,i}$	Reactive power generation level (Mvar)
v_i , δ_i	Module and angle of Bus *i* voltage phasor

Constant Parameters:

$p^b_{g\,i}$	Bilateral Active power generation level at bus i (MW)
$p_{d\,j}$	Pool Active power demand level at bus *j* (MW)
$p^b_{d\,j}$	Bilateral Active power demand level at bus *j* (MW)
$q_{d\,j}$	Reactive power demand level at bus *j* (Mvar)
R^{RU}	Amount of regulation up reserve required by the system (MW)
R^{RD}	Amount of regulation down reserve required by the system (MW)
R^{SR}	Amount of spinning reserve required by the system (MW)
R^{NS}	Amount of non spinning reserve required by the system (MW)
R^{RC}	Amount of complementary reserve required by the system (MW)
$RP^{RU}_{g\,i}$	10 minutes ramp rate for providing $ru_{g\,i}$ in MW/minute
$RP^{SNS}_{g\,i}$	10 minutes ramp rate for providing $sr_{g\,i}$ and $ns_{g\,i}$ in MW/minute

3 FORMULATION

In this section is presented the formulation of the combined market of energy (with pool and bilateral contracts) and reserve services. We consider a one hour auction with no inter-temporal constrains and suppose that enough support of reactive power is available to keep voltages close to nominal values.

3.1 Pool and Reserve Auction

The combined Pool and reserve auction determine awarded energy and reserve bids for a period of one hour. In this market low cost merit order lists for energy and reserve services are obtained by minimizing the following objective function.

$$Minimize \quad C_{energy} + C_{reserve} \tag{1}$$

Where,

$$C_{energy} = \sum_i C_i\,(p^p_{g\,i})$$

$$C_{reserve} = \{\sum_i C^{RU}_i(ru_{g\,i}) + \sum_i C^{SR}_i(sr_{g\,i})$$

$$+\sum_i C^{NS}_i(ns_{g\,i}) + \sum_i C^{RC}_i(rc_{g\,i}) + \sum_i C^{RD}_i(rd_{g\,i})\}$$

Cost bid[1] functions can be considered as continuous quadratic or piece-wise linear functions.

3.2 Transmission Network

From the Independent System Operator - ISO point of view, awarded bids and firm bilateral contracts must attend operation constraints imposed by the transmission system. Therefore, feasible solutions should belong to the set defined by constraints (2) to (7) for all buses *i* and where long term physical bilateral contracts and the pool demand are known quantities. Each demand j is considered with two energy components $p_{d\,j} = p^p_{d\,j} + p^b_{d\,j}$, and generator at bus *i* also has two energy components $p_{g\,i} = p^p_{g\,i} + p^b_{g\,i}$. The network load flow equations are represented by (2) and (3). Constraint (4) represents transmission line limit of active flow, (5) to (7) represent generation capacity limits and voltage limits. Vector V represents bus voltage modules and vector δ represents bus voltage phase angles. Both vectors have dimension equal to the total number of buses *n.*

$$p_{g\,i} - p_{d\,i} = p_i(\text{v},\delta), \quad \rightarrow \lambda_i \tag{2}$$

$$q_{g\,i} - q_{d\,i} = q_i(\text{v},\delta), \quad \rightarrow \lambda^q_i \tag{3}$$

$$-\,p^{\max}_{i\,j} \le p_{i\,j} \le p^{\max}_{i\,j} \tag{4}$$

$$p^{\min}_{g\,i} \le p_{g\,i} \le p^{\max}_{g\,i} \tag{5}$$

$$q^{\min}_{g\,i} \le q_{g\,i} \le q^{\max}_{g\,i} \tag{6}$$

$$v^{\min}_i \le v_i \le v^{\max}_i \tag{7}$$

Lambdas in (2) and (3) are Lagrange multipliers associ-

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[1] Under pay-as-bid, generators bid above their true cost. In this paper, generator cost is understood to mean bid cost.

ated to each constraint and represent nodal prices for active and reactive power.

3.3 Reserve Market characteristics

Generators agents can bid in five kinds of reserve services: Regulation up, Regulation Down, Spinning, Non-Spinning, and Replacement. The speed of response defines the quality of each service. Faster regulations services such as regulation up and down and spinning reserve are considered of higher quality than the others. In order to open possibilities for reducing costs, it is also allowed the possibility of substitution among reserve services. The substitution consists in allowing that services with better quality and lower cost can substitute services with lower quality and higher cost [1,7]. Hence, a feasible bid selection in the reserve market auction should belong to the set described by constraints (8) to (14).

$$R^{RU} \le \sum_{i=1}^{n} ru_{g\,i} \quad \to \lambda^{RU} \tag{8}$$

$$R^{RU} + R^{SR} \le \sum_{i=1}^{n} ru_{g\,i} + \sum_{i=1}^{n} sr_{g\,i} \quad \to \lambda^{SR} \tag{9}$$

$$R^{RU} + R^{SR} + R^{NS} \le \sum_{i=1}^{n} ru_{g\,i} + \sum_{i=1}^{n} sr_{g\,i} + \sum_{i=1}^{n} ns_{g\,i} \to \lambda^{NS} \tag{10}$$

$$R^{RC} \le \sum_{i=1}^{n} rc_{g\,i} \quad \to \lambda^{RC} \tag{11}$$

$$R^{RD} \le \sum_{i=1}^{n} rd_{g\,i} \quad \to \lambda^{RD} \tag{12}$$

$$ru_{g\,i} \ge 0, sr_{g\,i} \ge 0, ns_{g\,i} \ge 0, rc_{g\,i} \ge 0, rd_{g\,i} \ge 0 \tag{13}$$

$$ru_{g\,i} \le ru_i^{\max}, sr_{g\,i} \le sr_i^{\max}, ns_{g\,i} \le ns_i^{\max}, \\ rc_{g\,i} \le rc_i^{\max}, rd_{g\,i} \le rd_i^{\max} \tag{14}$$

Where, the estimated required system amounts of each reserve service are considered known and defined by the ISO before the market auction is performed. Lambda variables are Lagrange multipliers associated to each constraint. Upper limits in (14) are related to physical limits such as generators ramp rates and they are part of the information of the reserve bids.

3.4 Long Term Bilateral Contracts

Private long term bilateral contracts are considered as firm physical (not financial) contracts which are authorized and implemented by the ISO taking into consideration the reliability conditions of the transmission network. In a compact form, bilateral contracts can be grouped in a GD matrix where each coefficient $GD_{i\,j}$ represents the MW traded between generators at bus *i* and load at bus *j*. Therefore, the total amount of contracts supplied by generator *i* is,

$$p_{g\,i}^{b} = \sum_{j=1}^{n} GD_{i\,j} \tag{15}$$

In addition, the total amount of contracts supplying the demand at bus *j* is,

$$p_{d\,j}^{b} = \sum_{i=1}^{n} GD_{i\,j} \tag{16}$$

In this model is supposed that firm long-term bilateral contracts already exist at the moment of performing one auction [4]. Because of this, the already committed capacity with bilateral contracts imposes a constraint on the lower generation limit for generators participating in the pool market as shown in (17),

$$p_{g\,i}^{b} \le p_{g\,i} \le p_{g\,i}^{\max} \tag{17}$$

3.5 Reserve Availability of Generators Capacity

In the combined market, besides attending bilateral and pool loads, each generator *i* can also participate by bidding in several reserve services in the reserve market based on its availability. The awarded reserve bids should respect the operational capacity limits of each generator as described in (18) and (19).

$$p_{g\,i} + ru_{g\,i} + sr_{g\,i} + ns_{g\,i} \le p_{g\,i}^{\max} \tag{18}$$

$$-rd_{g\,i} + p_{g\,i} \ge p_{g\,i}^{b} \tag{19}$$

In addition, the ramp rate constraint MW/minute in a 10 minute time basis is considered through the following linear relation.

$$\frac{ru_{g\,i}}{RP_{g\,i}^{RU}} + \frac{rs_{g\,i} + ns_{g\,i}}{RP_{g\,i}^{SNS}} - 10 \le 0 \tag{20}$$

3.6 Marginal Price of Reserve Services

The Lagrangean function of the optimization problem described in (1) to (20) allows to obtain expressions for the market clearing prices of services based on the Lagrange Multipliers which represent the sensitivity of cost in terms of reserve requirements as shown in (21) to (25).

$$\frac{\partial L}{\partial R^{RU}} = \lambda^{RU} + \lambda^{SR} + \lambda^{NS} = MCP_{RU} \tag{21}$$

$$\frac{\partial L}{\partial R^{SR}} = \lambda^{SR} + \lambda^{NS} = MCP_{SR} \tag{22}$$

$$\frac{\partial L}{\partial R^{NS}} = \lambda^{NS} = MCP_{NS} \tag{23}$$

$$\frac{\partial L}{\partial R^{RC}} = \lambda^{RC} = MCP_{RC} \tag{24}$$

$$\frac{\partial L}{\partial R^{RD}} = \lambda^{RD} = MCP_{RD} \tag{25}$$

Since Lagrange multipliers are positive, this formulation ensures not reversal prices among services. In other words, $MCP_{RU} \ge MCP_{SR} \ge MCP_{NS}$ as discussed in [1].

4 PAY AS BID INCREMENTAL MODEL

The pay as bid version of this model is implemented through an integration process based on the Auman-Shapley technique [3,5] that takes into consideration the non-linear characteristic existing in the transmission network due to transmission losses and voltage behaviors. The integration process allows also to unbundling the use and prices of several services including pool dispatch, reserve availability and bilateral contracts. One iteration of the process is described as follows.

Step1: Bilateral Contracts Losses and Congestion

In this step, losses attributed to bilateral contracts due to the use of the transmission network are compensated in the Pool market. Initially, bilateral load is incremented by $dGD_{i\,j}$ while holding fixed the pool load and reserve requirements whose value at the very beginning are nil. In this step, the optimization problem minimizes costs of loss compensation due to bilateral contracts and possible congestion management. More precisely, the incremental optimization problem to be solved is defined by (1) to (20) considering for each load j an incremental variation $dp^{b}_{d\,j} = \sum_{i} dGD_{i\,j}$ while holding fixed pool loads $p^{p}_{d\,j}$ and reserve requirements R^{RU}, $R^{SR}, R^{NS}, R^{RC}, R^{RD}$. Calling the solution of this problem as $dp^{*}_{g\,i}$ then the contract incremental losses and congestion management are obtained through (26).

$$dp^{bpcl}_{gi} = dp^{*}_{g\,i} - \sum_{j=1}^{n} dGD_{i\,j} \qquad (26)$$

Step 2: Pool and Reserve Market

In this step, the incremental optimization problem (1) to (20) is solved incrementing pool loads j by $dp^{p}_{d\,j}$. Reserve requirements are incremented consecutively by $dR^{RU}, dR^{SR}, dR^{NS}, dR^{RC}$, dR^{RD} while holding fixed the bilateral contract loads. The solution of this optimization problem gives the awarded pool incremental generation levels, dp^{p}_{g}, and incremental reserve levels, $dru_i, dsr_i, dns_i, drc_i, drd_i$.

4.1 Integration Process

The integration process consists in performing alternatively step 1 and step 2 for small load increments following a uniform integration path from zero to the final value of the load, according to the "*t*" parameter such that $0 \le t \le 1$, as shown in (27).

$$X = \int_{t=0}^{1} dx(t) \qquad (27)$$

Vector dx is composed of variables $dp^{b}_{g\,i}$ $dp^{p}_{g\,i}$, $dp^{bpcl}_{g\,i}$, $dru_{g\,i}$, $dsr_{g\,i}$, $dns_{g\,i}, drd_{g\,i}$ and $drc_{g\,i}$.

5 REVENUES AND PAYMENTS

Economic indexes are presented incrementally in this section. Their corresponding final values are obtained by performing the previously described integration process.

5.1 Bilateral Contracts

Because bilateral contracts are negotiated in private, their prices are not available. Nevertheless, we adopt as a price for these contracts the corresponding incremental bilateral costs of generators i.e., $IC^{b}_{g\,i} = dC_{g\,i}(P^{b}_{gi}) \big/ dP_{gi}$.

Revenues: The generator i revenue due to only bilateral contracts is,

$$dc^{b}_{gi} = IC^{b}_{gi} \cdot \sum_{j=1}^{n} dGD_{i\,j} \qquad (28)$$

Payments: Load j pays the supplied bilateral contracts according to (29),

$$dc^{b}_{d\,j} = \sum_{i=1}^{n} IC^{b}_{i} \cdot dGD_{i\,j} \qquad (29)$$

Bilateral contracts should pay for losses and congestion management according to (30),

$$dc^{bcl}_{i\,j} = \sum_{i=1}^{n} (\lambda_j - \lambda_i) \cdot dGD_{i\,j} \qquad (30)$$

This amount could be split among contract parties in a 50/50 basis or other proportion; we adopt a split of 50/50 among contract parties. The corresponding payment of generator i for all its bilateral contracts losses and congestion is,

$$dc^{bcl}_{gi} = (1/2) \sum_{j=1}^{n} dc^{bcl}_{i\,j} \qquad (31)$$

Similarly, the payment of the bilateral load j is,

$$dc^{bcl}_{d\,j} = (1/2) \sum_{i=1}^{n} dc^{bcl}_{i\,j} \qquad (32)$$

5.2 Pool and Reserve

Revenues: Generator i has a revenue portfolio from participating in attending pool demand, bilateral loss compensation and congestion management as well as providing availability of reserve services given by (33) to (36).

$$d\hat{c}^{p}_{g\,i} = \lambda_i \cdot dp^{p}_{g\,i} \qquad (33)$$

$$dc^{bpcl}_{gi} = \lambda_i \cdot dp^{bpcl}_{g\,i} \qquad (34)$$

$$dc^{RU}_{g\,i} = MCP_{RU} \cdot dru_{g\,i} \qquad (35)$$

$$dc^{SR}_{g\,i} = MCP_{SR} \cdot dsr_{g\,i} \qquad (36)$$

Incremental revenues from other reserve services follow the same calculations.

Payments: Load j has a payment portfolio due to the use of pool demand including losses and reserve services following (37) to (39). In the case of reserve services, load j pays in a pro-rate manner as shown in (38) and (39).

$$d\hat{c}^{p}_{d\,j} = \lambda_j dp^{p}_{d\,j} \qquad (37)$$

$$dc^{RU}_{d\,j} = MCP_{RU} \cdot (\sum_{i=1}^{n} dru_{g\,i}) \cdot (p^{p}_{d\,j} + p^{b}_{d\,j}) / p_{d\,Total} \qquad (38)$$

$$dc^{SR}_{d\,j} = MCP_{SR} \cdot (\sum_{i=1}^{n} dsr_{g\,i}) \cdot (p^{p}_{d\,j} + p^{b}_{d\,j}) / p_{d\,Total}$$

(39)

Incremental load payments related to the other services are obtained in a similar way.

6 GENERATORS AND LOADS PORTFOLIOS

At the end of the integration process by using (27), we have the net portfolio revenue of generator *i* which is composed basically by three terms corresponding to bilateral, pool and reserve markets as follows,

$$c_{gi} = c_{gi}^{b} + c_{gi}^{p} + c_{gi}^{R} \tag{40}$$

Where,

$$c_{gi}^{R} = c_{gi}^{RU} + c_{gi}^{RD} + c_{gi}^{SR} + c_{gi}^{NS} + c_{gi}^{RC} \tag{41}$$

$$c_{gi}^{p} = \hat{c}_{gi}^{p} + c_{gi}^{bpcl} - c_{gi}^{bcl} \tag{42}$$

Likewise, the portfolio payment of load *j* is composed by three terms corresponding to bilateral, pool and reserve markets as follows,

$$c_{dj} = c_{dj}^{b} + c_{dj}^{p} + c_{dj}^{R} \tag{43}$$

Where,

$$c_{dj}^{R} = c_{dj}^{RU} + c_{dj}^{RD} + c_{dj}^{SR} + c_{dj}^{NS} + c_{dj}^{RC} \tag{44}$$

$$c_{dj}^{p} = \hat{c}_{dj}^{p} + c_{dj}^{bcl} \tag{45}$$

7 RECONCILIATION OF COSTS

Under the pay-as-bid scheme, the costs allocated to the loads and bilateral contracts perfectly match the generation cost components [5]. This characteristic is also verified in the reserve market at the end of the integration process by applying (27). For instance, in the case of the spinning reserve service we have,

$$\sum_{i=1}^{n} c_{gi}^{SR} = \sum_{j=1}^{n} c_{dj}^{SR} \tag{46}$$

8 NUMERICAL RESULTS

This section analyzes results of an IEEE 5-Bus system when the suggested pricing model is applied. Table 1 shows the network data in per unit in a base of 100 MVA and 200 kV. For simplicity, it is considered enough reactive support to keep bus voltages within limits. Table 2 describes generator bids with capacity limits and continuous bid functions c_{gi}. The only two reserve services required by the system are R^{RU}= 14.4 MW and R^{SR}= 40 MW whose sum represent a reserve margin of 5% of total load (i.,e., 54.4 MW). In order to simplify the amount of data, the generators bid functions for regulation up reserve service are considered half of the corresponding energy bids in Table 2 (i.e., $a_{g1}^{RU} = 10$ \$/MWh and $b_{g1}^{RU} = 0.02$ \$/MW^{2}h for the generator 1 reserve bid and similar for the other generators regulation up reserve bids). Spinning reserve bids for all generators are considered more expensive and equal to the corresponding energy bids. In order to simplify the observation, cases with transmission congestion and the influence of ramp constraints are not presented. The total fixed system load of 1088 MW is distributed among buses according with the following vector: $p_d = [34,85,119,323,527]^T$ (MW). This load is supplied by bilateral and pool markets in a proportion of 94% of bilateral contracts and 6% of pool load. Table 3 shows the corresponding bilateral contracts matrix, GD. In this case, the pool loads required at each bus are the coefficients of vector $p_d^p = [2,5,7,19,31]^T$ (MW). Bilateral tariffs are chosen by, $\pi_{ij}^{b} = dC_{gi}(P_{gi}^{b}) / dP_{gi}$ for all bilateral loads *j*.

From Bus	To Bus	r (pu)	x (pu)	b (pu)	p_{flow}^{max} (pu)
1	2	0.0147	0.168	0.138	3.00
1	4	0.0108	0.126	0.102	3.55
2	3	0.0185	0.210	0.185	3.00
3	4	0.0294	0.336	0.296	3.00
3	5	0.0221	0.252	0.213	3.00
4	5	0.0108	0.126	0.104	4.50
2	4	0.0105	0.130	0.100	3.60

Table 1: Network Data.

$$C_{gi}(p_{gi}) = c_{0i} + a_i \cdot p_{gi} + 0.5 \cdot b_i \cdot p_{gi}^2 \quad (\$/h)$$

Bus	p_g^{min} (MW)	p_g^{max} (MW)	c_0 (\$/h)	a (\$/MWh)	b (\$/MW2h)
1	0	460	0	20	0.040
2	0	500	0	21	0.030
3	0	500	0	25	0.045
4	0	500	0	56	0.040
5	0	500	0	57	0.040

Table 2: Generators Energy Bids.

Bus		Loads (MW)					
		1	2	3	4	5	Total
Generators (MW)	1	32	48	32	144	160	416
	2	0	32	32	112	240	416
	3	0	0	48	48	96	192
	4	0	0	0	0	0	0
	5	0	0	0	0	0	0
	Total	32	80	112	304	496	

Table 3: Bilateral Contracts (MW).

The first case is presented in Table 4. The first six rows illustrate the three market dispatches. Generators 1 and 2 are heavily committed with bilateral contracts (83% of their capacity) imposing an out of merit operation for pool market. The economic dispatch only awards pool bids of generator 2 and 3 for attending pool load and transmission losses of 43.63 MW. Only the low cost generator 2 participates in the three markets. Due to the low cost of the regulation up service of generators, the more expensive spinning reserve bids are not accepted. Rows 10 to 15 show economic indexes such as nodal prices, incremental costs and average prices of services of the three markets.

Bus	1	2	3	4	5	Total
P_g^b	416.0	416.0	192.0	0.0	0.0	1024.0
P_g^p	0.0	47.8	59.8	0.0	0.0	107.6
P_g	416.0	463.8	251.8	0.0	0.0	1131.6
ru_g	37.6	16.8	0.0	0.0	0.0	54.4
sr_g	0.0	0.0	0.0	0.0	0.0	0.0
Total R	37.6	16.8	0.0	0.0	0.0	54.4
${}_*P_d^b$	32.0	80.0	112.0	304.0	496.0	1024.0
${}_*P_d^p$	2.0	5.0	7.0	19.0	31.0	64.0
${}_*P_d$	34.0	85.0	119.0	323.0	527.0	1088.0
λ	34.5	34.9	36.3	37.1	39.9	-
IC_g	36.6	34.9	36.3	56.0	57.0	-
${}_*\pi_g^b$	36.6	33.5	33.6	56.0	57.0	-
C_g^b/P_g^b	28.3	27.2	29.3	-	-	-
C_g^p/P_g^p	-	21.7	26.3	-	-	-
C_g^{RU}/ru_g	10.4	10.6	-	-	-	-
c_g^b	11781	11331	5629.4	0.0	0.0	28742
c_g^p	0.0	1039.0	1575.3	0.0	0.0	2614.3
c_g^R	390.1	178.5	0.0	0.0	0.0	568.7
Total Rev.	12171	12549	7204.7	0.0	0.0	31925
c_d^b	898.2	2245.5	3143.7	8532.9	13922	28742
c_d^p	81.7	204.2	285.9	776.1	1266.3	2614.3
c_d^R	17.8	44.4	62.2	168.8	275.4	568.7
Total Pay.	997.7	2494.2	3491.8	9477.8	15463	31925

Table 4: Energy and Reserve market dispacht through the integrated PAB approach with revenues and payments. Bilateral Contracts participation is equal to 94% of total load. Power in MW, λ, π_g^b and average cost in \$/MWh. Rows with * contain given data.

Due to the fact that the bilateral contracts only exist for the more economical generators, their average bilateral prices are lower than their corresponding incremental costs. Average prices for energy are lower than marginal prices with average prices for reserve being slightly lower than the corresponding marginal clearing price, MCP_{ru}, which is equal to 10.8\$/MWh.

Bus	1	2	3	4	5	Total
P_g^b	460.0	500.0	64.0	0.0	0.0	1024.0
P_g^p	0.0	0.0	51.3	68.1	0.0	119.4
P_g	460.0	500.0	115.3	68.1	0.0	1143.4
ru_g	0.0	0.0	54.4	0.0	0.0	54.4
sr_g	0.0	0.0	0.0	0.0	0.0	0.0
Total R	0.0	0.0	54.4	0.0	0.0	54.4
${}_*P_d^b$	32.0	80.0	112.0	282.0	518.0	1024.0
${}_*P_d^p$	2.0	5.0	7.0	41.0	9.0	64.0
${}_*P_d$	34.0	85.0	119.0	323.0	527.0	1088.0
λ	-103.7	-3.7	30.2	58.7	53.0	-
IC_g	38.4	36.0	30.2	58.7	57.0	-
${}_*\pi_g^b$	38.4	36.0	27.9	56.0	57.0	-
C_g^b/P_g^b	29.2	28.5	26.4	-	-	-
C_g^p/P_g^p	-	-	26.2	57.4	-	-
C_g^{RU}/ru_g	-	-	13.1	-	-	-
c_g^b	13432	14250	1692.2	0.0	0.0	29374
c_g^p	0.0	0.0	1488.2	3909.0	0.0	5397.2
c_g^R	0.0	0.0	713.3	0.0	0.0	713.3
Total Rev.	13432	14250	3893.7	3909.0	0.0	35484
c_d^b	917.9	2294.9	3212.8	8089.4	14859	29374
c_d^p	168.7	421.7	590.3	3457.6	759.0	5397.3
c_d^R	22.3	55.7	78.0	211.8	345.5	713.3
Total Pay.	1108.9	2772.3	3881.1	11758	15463	35484

Table 5: Energy and Reserve market dispacht through the integrated PAB approach with revenues and payments. Bilateral Contracts participation is equal to 94% of total load. Bilateral Contracts of Generator 1 are now equal to 460 and for Generator 2 is 500 MW. Power in MW, λ, π_g^b and average cost in \$/MWh. Rows with * contain given data.

Rows 16 to 23 show the agents participation in the three markets indicating generators revenue components and load payment components. The highest revenue belongs to the cheap generator at bus 2 with 12,549\$ (who obtains revenues from the three markets) and the highest payment is from load at bus 5 with 15,463\$. The reconciliation of revenues and payments for energy (bilateral and pool) and reserve can be observed by comparing the coefficients in the last column of these rows (related to revenues and payments) which are identical and consequently no Merchandising Surplus (MS) is produced. If the same generator bids were used in a marginal pricing approach, the total cost of operation is 40,296.1 \$/h which represents an increase of 26.2% in relation to the total cost of 31,925.3 \$/h obtained by the PAB approach. The last numerical example presents results of a case with the same total load and proportion of bilateral, pool and reserve requirements as the previous case but with different distribution of bilateral contracts. The following coefficients of the bilateral contracts matrix change: GD_{14}=144+22=166, GD_{15}= 160+22 = 182, GD_{25}= 240+84 = 324, GD_{34}=48-44=4, GD_{35}=96-84=12. Table 5 shows that with this new distribution of bilateral contracts the more economical generators 1 and 2 allocate all of its capacity to supply only bilateral load. The other generators have capacity for attending pool demand and reserve requirements. In the Pool market only generator 3 and generator 4 are awarded for supplying pool load and transmission losses of 55.4 MW. In the reserve market only generator 3 is awarded for supplying the total system reserve requirement of 54.4 MW. Due to the out of merit operation imposed by the high level of bilateral contracts, nodal prices λ under marginal pricing are quite volatile in contrast with the more stable average prices by PAB (rows 13 to 15). Negative values of nodal prices (-103.7 $/MWh at bus 1 and -3.7 $/MWh at bus 2), under marginal pricing, gives a signal that an increase in pool demand at that buses decreases the total cost. In relation to the previous case, average prices have only a slight increase. Comparing tables 4 and 5 it is also possible to observe that in spite of the fact generator 3 is participating in all three markets its total revenue (3,893.6 $/h) becomes lower than in the previous case (7,204.7 $/h) due to its reduction in supplying the bilateral market. Moreover, it is possible to observe that load payments in the pool market increase significantly (more than 100%) due to the participation of generator 4 in this market. Loads also pay more for reserve in this case (an increase of 25.4%). Total cost increased 11% . With a marginal pricing approach total cost is 16.5 % bigger (41,325.9 $/h).

Numerical cases show that even considering reserve service bids independent of the generators energy bids in a joint market, the resulting reserve prices depend on the committed capacity allocated to supply the long term bilateral contracts and pool demand and associated prices. This interaction represents economic signals for both energy and reserve markets allowing generators to be able to estimate and plan their opportunity cost and portfolios in a consistent way because of the stability of prices and adequacy obtained in the three markets by using PAB. The more stable and adequate behavior (if compared with the nodal prices under marginal pricing) is verified by observing different firm bilateral contract distributions and represents an advantage for market purposes. Moreover, the pricing model always allows obtaining the reconciliation of revenue and payment portfolios in several operation conditions. In all the numerical cases presented the integration error is not greater than 0.001 and this precision is obtained within a number of 17 integration steps. It is also worth to mention that the classical marginal pricing approach is obtained in this formulation when only one integration step is performed.

9 CONCLUSION

In this paper a pricing model is presented with the following characteristics: i) incorporation of bilateral, pool, and reserve markets in a joint market of services; ii) the combined market allows assessing the impact of electricity products interactions on the operation and consequently on prices; iii) Allows to compare the pay-as-bid pricing and uniform pricing approaches in several scenarios; iv) Market agents can obtain detailed portfolios in terms of revenues and payments for awarded bids; v) ensures the reconciliation between payments and revenues with a reasonable price stability; vi) Allows to obtain economic signals for estimating opportunity costs of electricity products; vii) Allows the possibility of testing several operating scenarios with price strategies in order to evaluate the impact on agents portfolios.

The model considers long term bilateral contracts and the transmission system operation in detail including generation and transmission capacities, transmission losses, voltage and reactive limits. The characteristics of this model make it attractive for analyzing the impact of several operation scenarios and bid strategies on the agents portfolios based on historic data. Further research is under development for studying in detail the impact of fault uncertainty, inter-temporal constraints and the model simplification for application on large power systems.

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